A Rigorous Composite-Inflow-Performance Relationship Model for Multilateral Wells

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Summary

Use of multilateral wells for oil and gas production has gained strong momentum in the past 5 years. However, most of the multilateral wells do not deliver hydrocarbon fluids at expected production rates. One of the reasons for this is that the well planners overestimate the productivity of wells by using inaccurate methods for predicting composite-inflow-performance relationship (IPR) of well laterals. A more-accurate method for predicting composite IPR of multilateral wells is highly desirable. This paper fills the gap.

Starting from terms familiar to petroleum engineers, a general well model was developed with consideration of reservoir-wellbore crossflow for lateral IPR and coupling of fluid flow from individual laterals to the main wellbore. The model allows different IPRs of laterals and permits crossflow between laterals. Pressure losses in the vertical-, curvic- and horizontal-hole sections are rigorously considered. Oil and gas wells are treated differently. The modified Hagedorn-Brown correlation is used for modeling the flow in the vertical sections, and the Beggs and Brill correlation is used for the curvic and horizontal sections for oil wells. The Cullender and Smith method was used for modeling the flow in gas wells. A computer simulator was developed based on the model for predicting multilateral-well production rate. Case studies have indicated excellent accuracy of the computer model. This work provides petroleum engineers a reliable and user-friendly tool for designing and evaluating multilateral wells.

Introduction

Although oil production by use of multiple-drainholes was reported in the 1960s (Borisov 1964), popular applications of multilateral wells for producing oil and gas began in the early 1990s (Hardman 1993) after modern horizontal drilling technology was developed. Salas, Clifford and Jenkins (1996) identified eight categories of main potential applications of multilateral wells. Vij, Narasaiah, Walia et al. (1998) provided an overview of the multilateral technology and its limitations.

Raghavan and Joshi (1993) presented an analytical solution of well productivity for symmetric horizontal radials defined as horizontal drainholes of equal length kicked off from the same depth in symmetrical directions. The result was an inflow equation (i.e., the effect of wellbore flow from sandface to the common kick-off point was not considered). With a semianalytical solution, Retnanto and Economides (1996) demonstrated the benefits of using symmetrical multilateral wells in low- to medium-permeability reservoirs. Again, only lateral-inflow performance was considered. Larsen (2006) presented closed-form expressions of skin factors and productivity indices of radial-symmetric multilateral wells. Wellbore hydraulics was not considered. Salas, Clifford and Jenkins (1996) presented an IPR model for multilateral wells with the total skin factor lumping the effects of reservoir homogeneity and other factors. Wellbore hydraulics was also neglected. Wolfsteiner, Durlofsky and Aziz (2000) developed a general and sophisticated model for productivity of nonconventional wells in heterogeneous reservoirs. Wellbore hydraulics was not included in the model. Yildiz (2002) presented a similar solution that also neglected the effect of wellbore hydraulics. Yildiz (2005) compared his 3D multilateral-well model with data from an electrolytic model and the Salas, Clifford and Jenkin's (1996) model. Good agreements were observed. Smith, Tweedie and Gallivan (1997) addressed the importance of coupling the effects of fluid flow through perforations and wellbore hydraulics in reservoir simulation for multilateral-well economics evaluation. Permadi, Wibowo and Permadi (1998) investigated the effect of wellbore hydraulics on inflow performance of a stacked dual-lateral well, assuming single-phase flow in the wellbore. Ouyang and Aziz (1998) presented a simplified approach to coupling wellbore hydraulics and reservoir inflow for arbitrary well configurations. Chen, Zhu and Hill (2000) presented another model of multilateral-well deliverability by considering segmented horizontal holes and single-phase liquid flow in the horizontal sections of the well. Kamkom and Zhu (2005) applied Vogel's (1968) two-phase flow correlation to multilateral wells. The absolute open flow (AOF) was determined by use of Babu and Odeh's (1989) horizontal-well IPR model. The wellbore hydraulics was modeled with the correlation of Beggs and Brill (1973). Kamkom and Zhu's model is valid for reservoir pressures being lower than the bubblepoint pressure. Although the hydraulics in the horizontal branches was considered numerically, this is the first multilateral-well model that considers hydraulics in all the wellbore sections. Ouyang and Huang (2005) history-matched the oil production from a two-lateral well by coupling reservoir inflow and wellbore hydraulics in a numerical simulator. The paper does not describe how the hydraulics in the horizontal wellbore was simulated.

In summary, most of the multilateral-well productivity models were derived on the basis of mathematical analyses of fluid flow in the reservoir side, leaving the flow inside the horizontal- and curvic-wellbore sections as a part of the outflow performance of the well. A few studies have considered the wellbore hydraulics in the horizontal section by assuming single-phase flow. Only one study by Kamkom and Zhu (2005) addressed the wellbore hydraulics in the horizontal and curvic sections. However, in reality, multilaterals are not kicked off at the same point (i.e., kick-off points are separated by several vertical sections). The hydraulics in these vertical sections is expected to have more impact on well productivity than the hydraulics in the horizontal sections. This is because the hydrostatic-pressure components that do not exist in the horizontal sections will reduce well deliverability. No literature has been found to address this issue. A more-accurate model considering hydraulics in all the wellbore sections is highly demanded.

In this study, we define the upper-most conjunction as the end of inflow system. We have derived a composite-IPR model by rigorously coupling the wellbore hydraulics inside the well sections (i.e., vertical, curvic and horizontal intervals) and the inflow models of all the laterals (lateral IPR). Because the outflow from the upper-most conjunction is a well-known subject, it is excluded from the scope of this paper although a multiphase-outflow model has been incorporated in our computer program.

The composite IPR of both multilateral oil and gas wells were developed in this study. Numerical results of these composite-IPR models were compared with that of field wells, and a very good agreement was observed. This paper provides petroleum engineers with a simple and accurate method for predicting and evaluating performance of multilateral wells.

Reservoir-Horizontal Wellbore Crossflow

Fig. 1 shows a sketch of a horizontal drainhole. An analytical-IPR model was derived in this study for oil wells, coupling reservoir inflow and frictional-pressure loss in the horizontal wellbore. The procedure used for deriving the analytical model is similar to that

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This paper (SPE 100923) was accepted for presentation at the 2006 SPE Annual Technical Conference and Exhibition, San Antonio, Texas, 24–27 September 2006. Original manuscript received for review 29 May 2006. Revised manuscript received 28 March 2007. Paper peer approved 9 October 2007.

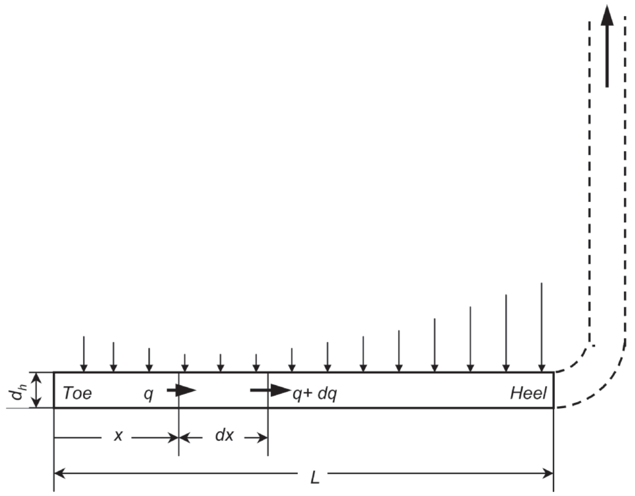

Fig. 1—Sketch of a horizontal drainhole.

employed by Dikken's (1990) model. Dikken's boundary condition that the drawdown derivative is equal to zero at the toe was replaced in this study by the more-realistic boundary condition that the drawdown derivative is equal to zero at any point where the drawdown is zero. Derivation of the new model is available from the authors upon request. Resultant equations are summarized in this section. Units of variables are explained in the Nomenclature section.

Consider a steady-state-flow situation. It is expected that the volumetric-flow rate of the production fluid will increase as the fluid flows from the toe to the heel as a result of the fluid inflow from reservoir. When the flow rate reaches a critical value at a critical distance from the toe, the flow pattern changes from laminar flow to turbulent flow after a transition interval. Assuming the critical Reynolds number is 2,000, an expression for the critical-flow rate (q_c) can be obtained as follows:

$$q_c = 1351.34\,\frac{\mu_o d_h}{\rho}, \quad \dots\dots (1)$$

where μ_o, ρ and d_h are oil viscosity, density and wellbore diameter, respectively. The critical distance (x_c) can be expressed as

$$x_c = \frac{1}{\sqrt{c}} \ln\left[1 + 1351.34\,\frac{\mu_o d_h e^{\sqrt{cL}}\sqrt{c}}{\rho J_{sp}(p_r - p_{wf})}\right], \quad \dots\dots (2)$$

where

$$c = \frac{7.96 \times 10^{-6} \mu_o J_{sp}}{d_h^4}, \quad \dots\dots (3)$$

and where L, J_{sp}, p_r and p_{wf} are horizontal-wellbore length, specific-productivity index, reservoir pressure and the flowing-heel pressure, respectively. If the drainhole length is less than the critical distance ($L<x_c$), then the total flow rate from the drainhole is in laminar flow and is expressed as

$$q = \frac{J_{sp}(p_r - p_{wf})(1 - e^{-\sqrt{cL}})}{\sqrt{c}}. \quad \dots\dots (4)$$

If $L>x_c$, the total flow rate from the drainhole is contributed by both the laminar and turbulent flow intervals. It is expressed as

$$q = q_c + \frac{J_{sp}}{2b}\left[\frac{1}{(a + bx_c)^2} - \frac{1}{(a + bL)^2}\right], \quad \dots\dots (5)$$

where

$$a = \frac{1}{\sqrt[3]{p_r - p_{wf}}} + 0.2752 C^{\frac{2}{3}} L, \quad \dots\dots (6)$$

$$b = -0.2752 C^{\frac{2}{3}} \quad \dots\dots (7)$$

and

$$C = 0.001763 J_{sp} \sqrt{\frac{f_f \rho}{d_h^5}}, \quad \dots\dots (8)$$

where the Fanning friction factor (f_f) can be evaluated through the correlation presented by Chen (1979). Because the Reynolds number is flow-rate dependent, a trial-and-error procedure has to be employed to solve q in Eq. 5.

A similar mathematical model for horizontal gas wells has also been developed. It is not presented in this paper because of length consideration, but is available from the author upon request.

Description of Composite-IPR Model

Consider the multilateral well depicted in **Fig. 2.** Suppose the well has n laterals and each lateral consists of three sections: horizontal, curvic and vertical. Let L_i, R_i and H_i denote the length of the horizontal section, radius of curvature of the curvic section and length of the vertical section of lateral i, respectively. The IPR of the laterals, Eq. 5, can be expressed as follows:

$$q_i = f_{Li}(p_{wf_i}) \qquad i = 1, 2, \dots, n, \quad \dots\dots (9)$$

where q_i is the production rate from lateral i, f_{Li} is inflow performance function of the horizontal section of lateral i, and p_{wf_i} is flowing heel pressure in lateral i. The fluid flow in the curvic sections can be described by

$$p_{wf_i} = f_{Ri}(p_{kf_i}, q_i) \qquad i = 1, 2, \dots, n, \quad \dots\dots (10)$$

where f_{Ri} is flow-performance function of the curvic section of lateral i and p_{kf_i} is the flowing pressure at the kick-out point of lateral i. The fluid flow in the vertical sections may be described by

$$p_{kf_i} = f_{hi}\left(p_{hf_{i+1}}, \sum_{j=1}^{i} q_j\right) \qquad i = 1, 2, \dots, n-1, \quad \dots\dots (11)$$

where f_{hi} is flow-performance function of the vertical section of lateral i and p_{hf_i} is the flowing pressure at the top of vertical section of lateral i. The following relation holds true at the conjunction points:

$$p_{kf_i} = p_{hf_i} \qquad i = 1, 2, \dots, n. \quad \dots\dots (12)$$

Equation groups 9 through 12 contain ($4n-1$) equations. For a given flowing pressure p_{hf_n} at the top of the vertical section of lateral n, the following ($4n-1$) unknowns can be solved from the ($4n-1$) equations. They are: $q_1, q_2, \dots q_n$; $p_{wf_1}, p_{wf_2}, \dots p_{wf_n}$ $p_{kf_1}, p_{kf_2}, \dots p_{kf_n}$; and $p_{hf_1}, p_{hf_2}, \dots p_{hf_{n-1}}$. Then the production rate of the multilateral well at a given pressure p_{hf_n} at the upper-most conjunction can be determined by

$$q = \sum_{i=1}^{n} q_i. \quad \dots\dots (13)$$

Thus, the composite IPR is expressed as

$$q = f(p_{hf_n}). \quad \dots\dots (14)$$

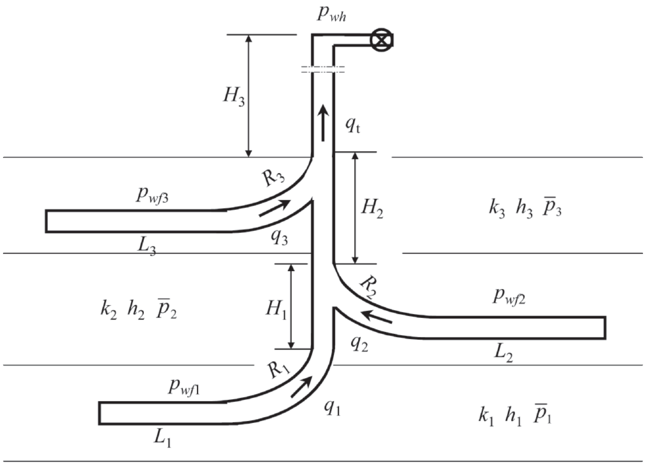

Fig. 2 —Nomenclature for a multilateral well.

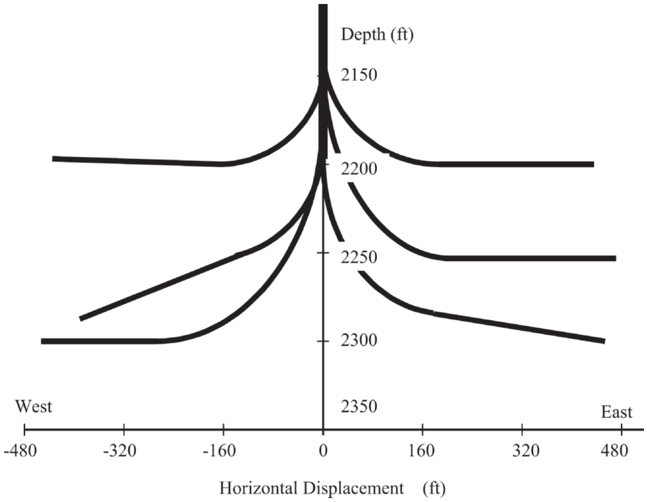

Fig. 3—Schematic of a six-branch multilateral well.

It should be noted that the composite-IPR model described here is general. If the vertical section of the top lateral is the production string (production through tubing and/or casing), then p_{hf_n} will be the flowing wellhead pressure. In this case, the relation expression (Eq. 14) represents wellhead-performance relationship (WPR).

It is understood that the lateral-IPR function f_{Li} can be defined differently, depending on reservoir type (gas or oil). Eq. 5 was used for oil wells, and Furui, Zhu and Hill's (2003) model was numerically applied to each segment of the horizontal sections for gas wells.

Solution Procedure

Consider the multilateral well depicted in **Fig. 3.** Suppose the well has *n* laterals counted from bottom up and each lateral consists of three sections: horizontal, curvic and vertical. The composite IPR of the multilateral well is defined in this paper as the relationship between the well-production rate q_t and the pressure p_{hf_n} above the upper-most conjunction of multilaterals.

1. For a given pressure at the exit of the vertical section of lateral *n*, p_{hf_n}, one starts well-IPR analysis with an assumed value of the total well-flow rate, q_t. The pressure at the kick-out point of lateral *n*, p_{kf_n}, can be calculated by employing vertical-lift performance (VLP):

$$p_{kf_n} = f_{h_n}(p_{hf_n}, q_t). \tag{15}$$

2. After p_{kf_n} is calculated with the assumed q_t value, the flowing heel pressure in lateral *n*, p_{wf_n}, can be estimated by combining the following curvic-section VLP formula and lateral-IPR formula:

$$p_{wf_n} = f_{R_n}(p_{kf_n}, q_n) \tag{16}$$

and

$$q_n = J_n(p_{rn} - p_{wf_n}). \tag{17}$$

Substitute Eq. 17 into Eq. 16 to obtain:

$$p_{wf_n} = f_{R_n}[p_{kf_n}, J_n(p_{rn} - p_{wf_n})]. \tag{18}$$

There is only one unknown, p_{wf_n}, in Eq. 18. A trial-and-error method is used to solve Eq. 18 for the flowing heel pressure in lateral *n*, p_{wf_n}.

3. Once p_{wf_n} is calculated with the assumed q_t value, production rate from lateral *n*, q_n can be calculated with Eq. 17.

4. The flowing pressure at the kick-out point of lateral $n-1$, $p_{kf_{n-1}}$, can be calculated with VLP of the vertical section:

$$p_{kf_{n-1}} = f_{h_{n-1}}(p_{kf_n}, q_t - q_n). \tag{19}$$

5. After the $p_{kf_{n-1}}$ is calculated with the assumed q_t value, the flowing heel pressure in lateral $n-1$, $p_{wf_{n-1}}$ can be estimated by combining the curvic-section VLP formula and lateral-IPR formula:

$$p_{wf_{n-1}} = f_{R_{n-1}}(p_{kf_{n-1}}, q_{n-1}) \tag{20}$$

and

$$q_{n-1} = J_{n-1}(p_{rn-1} - p_{wf_{n-1}}). \tag{21}$$

Substitute Eq. 21 into Eq. 20 to get

$$p_{wf_{n-1}} = f_{R_{n-1}}[p_{kf_{n-1}}, J_{n-1}(p_{rn-1} - p_{wf_{n-1}})], \tag{22}$$

which can be solved with a trial-and-error method for $p_{wf_{n-1}}$.

TABLE 1—RESERVOIR PROPERTIES OF THE TWO LAYERS (CASE 1)

Reservoir Property	Layer 1	Layer 2
Thickness (ft)	44	22
Porosity	0.23	0.22
Permeability (md)	144	144
Fluid viscosity (cp)	4.6	4.6
Formation volume factor (rb/stb)	1.14	1.14
Solution GOR (scf/stb)	212	212
Total compressibility (psi^{-1})	0.00001	0.00001
Reservoir pressure (psia)	1885	1900

6. Once $p_{wf_{n-1}}$ is calculated with the assumed q_t value, the production rate from lateral $n-1$, q_{n-1} can be calculated using Eq. 22.

7. Repeating steps 4 through 6 allows calculating the flowing pressure at the kick-out point of lateral $n-2$, $p_{kf_{n-2}}$; flowing heel pressure in lateral $n-2$, $p_{wf_{n-2}}$; and production rate from lateral $n-2$, q_{n-2}.

8. By use of the same procedure, one can calculate the flowing pressure at the kick-out point, p_{kf_i}; flowing heel pressure, p_{wf_i}; and production rate, q_i, for lateral *i*, $i=1, 2, \ldots, n-3$.

9. With calculated values of the individual production rates from laterals $q_1, q_2, \ldots q_n$, one can compare $q_1+q_2+\ldots+q_n$ with the assumed total flow rate, q_t. If the $q_1+q_2+\ldots+q_n-q_t>$tolerance, use $q_1+q_2+\ldots+q_n$ as new assumption for total flow rate, and repeat steps 1 through 8. If $q_1+q_2+\ldots+q_n-q_t<$tolerance, exit the loop and summarize the calculated values of $q_1, q_2, \ldots q_n$; $p_{wf_1}, p_{wf_2}, \ldots p_{wf_n}$; and $p_{kf_1}, p_{kf_2}, \ldots p_{kf_n}$.

10. Steps 1 through 9 yield one converged production rate q_t value for a given value of pressure p_{hf_n} above the upper-most conjunction of multilaterals. To construct composite IPR of the well, one needs to change p_{hf_n} values and repeat steps 1 through 9 to get the corresponding values of production rate, q_t. Then, a composite-IPR curve can be obtained by plotting p_{hf_n} vs. q_t.

Model Comparison and Verification

It is quite difficult to find clean data to verify composite IPR models for multilateral wells with more than two laterals. Field data from a dual-lateral well and a six-lateral well are used for the purpose of model verification in this paper.

Case 1. Karakas, Yokoyama and Arima (1991) presented a field case of a dual-lateral well drilled in a depleted reservoir. Because of the unfavorable fluid properties (viscous oil with little solution gas), a vertical well was kicked off to penetrate two layers with dual laterals. Reservoir properties of the two layers are summarized in **Table 1.** Permeabilities of the two layers were estimated from a pressure-transient test performed on the well after a hydraulic fracturing and immediately before the two lateral drainholes were drilled. Properties of the drainholes are presented in **Table 2,** in which the drainhole-productivity-index values were

TABLE 2—PROPERTIES OF DRAINHOLES PENETRATING TWO LAYERS (CASE 1)

Drainhole Property	Drainhole 1	Drainhole 2
Length (ft)	677	767
Radius (ft)	0.25	0.25
Productivity Index (stb/d-psi)	5.0218	2.5109

TABLE 3—SUMMARY OF PRODUCTION RATES (CASE 1)

Data Source	Drainhole 1	Drainhole 2	Well	Error (%)
Simple composite IPR	1,306	690	1,996	11.66
Kamkom-Zhu's model			1,928	7.86
This work	1,237	648	1,839	2.85
Test data			1,788	0.00

estimated on the basis of results from two pressure-transient tests carried out after each drainhole was drilled. The dual-lateral well was tested at a junction pressure of 1,625 psia, giving an oil-production rate of 1,788 STB/D. This production rate was compared with the results from a simple composite-IPR model, Kamkom and Zhu's (2005) model and the new model presented in this paper.

The simple composite-IPR model neglects the hydraulics in the horizontal, curvic and vertical sections below the upper-most junction point. This model predicts the total production by use of

$$q_t = \sum_{i=1}^{n} J_i(p_r - p_{wf}). \quad \ldots\ldots (23)$$

For the given dual-lateral well, this model yields

$$q_t = (5.0218)(1885 - 1625) + (2.5109)(1900 - 1625)$$
$$= 1996 \text{ stb/day},$$

which is 11.66% higher than the measured production rate of 1,788 STB/D.

Kamkom and Zhu's (2005) model considers wellbore hydraulics in the horizontal and the curvic sections, but not the vertical sections between laterals. It is expected that this model will overestimate deliverability of wells that have significant lengths of vertical-hole sections between the laterals' kick-off points. When applied to the given case, the model predicts a production rate of 1,928 STB/D, which is 7.86% greater than the measured production rate of 1,788 STB/D.

The model presented in this paper considers wellbore hydraulics in all the horizontal, curvic and vertical sections. When applied to the given case, the model predicts a production rate of 1,838 STB/D, which is 2.85% greater than the measured production rate of 1,788 STB/D. A summary of the model results is shown in **Table 3.**

Case 2. Another field case was presented by Lee, Brandao and Sotomayor et al. (2003) and Lage, Sotomayor and Vargas et al. (2003). The sandstone/conglomerate reservoir is at an average depth of approximately 700 m (2,300 ft). The net pay is approximately 100 m (330 ft), with a porosity of 15% and permability of 100 md. The oil has 22°API gravity with a viscosity of 30 to 700 mPa-s (30 to 700 cp) at the reservoir temperature of 50°C (122°F). The reservoir pressure was tested to be 741 psi at depth of 712 m (2,335 ft).

A six-branch multilateral well was drilled underbalanced to recover the heavy oil from an isolated area of the reservoir. A schematic of the well trajectory is shown in **Fig. 3.** All laterals were drilled out a 7-in. cased hole with 4-¾-in. bits and completed by dropping 2-7/8-in. perforated liners. **Table 4** summarizes lateral data based on Lage, Sotomayor and Vargas et al.'s (2003) paper.

The well was completed with a sucker-rod pump installed at the bottom of a 3-1/2-in. tubing string against the pay zone. The initial liquid-production rate of the well was 849 STB/D (698 STB/D of oil and 151 STB/D of water). **Table 5** presents a summary of branch-production rates calculated by the mathematical model. It shows a well production rate of 897 STB/D (735 STB/D of oil and 162 STB/D of water) against a pump-suction pressure of 554 psia. This calculated liquid production rate is 5.7% greater than the measured value of 849 STB/D.

Applications

The new model can be used for predicting deliverability and performance of multilateral wells. Two generic cases are studied in this section to show capability of the model. Case A is a study of an oil well with three laterals. Case B is a study of a gas well with four laterals. Each lateral penetrates only one reservoir layer. The reservoir layers are isolated from each other and only connected at the conjunctions of wellbores. Each reservoir layer has its own fluid and rock properties.

Fluid and rock properties and wellbore geometry are listed in **Table 6** for Case A. Fluid properties were evaluated at reservoir conditions. Because reservoir pressures and layer depths are different, it is expected that lateral crossflow in the well should occur when the wellhead pressure is higher than a certain value. To investigate this effect, it is necessary to change wellhead pressure and calculate corresponding total well-production and individual lateral-flow rates. This was done by use of a spreadsheet program. The results are shown in **Table 7,** which indicates that when the wellhead pressure is increased to 450 psi, the flow rate from layer 3 will be negative (i.e., this layer will become a thief zone). When the wellhead pressure is increased to 550 psi, the flow rate from layer 2 will be negative (i.e., this layer will also become a thief zone).

Fluid and rock properties and wellbore geometry are listed in **Table 8** for Case B. Again, fluid properties were evaluated at reservoir conditions. To investigate the effect of wellhead pressure on lateral crossflow, it is necessary to change wellhead pressure and calculate corresponding total well-production and individual lateral-flow rates. This was done by use of a spreadsheet program for gas wells. The results are shown in **Table 9** and plotted in **Fig. 4.** They indicate that when the wellhead pressure is increased to more than 1,800 psi, the flow rate from layer 4 will be negative (i.e., this layer will become a thief zone). When the wellhead pressure is increased to 1,960 psi, the flow rate from layer 3 will be negative (i.e., this layer will also become a thief zone).

TABLE 4—WELL BRANCH AND INFLOW PERFORMANCE DATA (CASE 2)

Lateral number	1	2	3	4	5	6
Radius of curvature (ft)	249	177	167	216	348	180
Maximum inclination (deg)	86	73	91.2	89.5	92.2	93.3
Total lateral length (ft)	604	600	636	659	462	459
Total exposure (ft)	341	430	272	495	338	410
Reservoir pressure (psi)	742	715	688	729	732	692
Productivity index (stb/d-psi)	0.03	0.14	0.22	0.20	0.24	0.23

TABLE 5—SUMMARY OF CALCULATED PRODUCTION RATES (CASE 2)

Lateral no.	1	2	3	4	5	6	Total
Junction pressure (psi)	609	604	592	587	577	555	
Heel pressure (psi)	710	676	660	675	717	628	
Oil rate (STB/D)	93	130	77	187	44	204	735
Water rate (STB/D)	20	29	17	41	10	45	162

Conclusions

A rigorous mathematical model has been developed in this study for predicting deliverability of multilateral wells. The model couples inflow performance of each lateral with hydraulics in horizontal-, curvic- and vertical-wellbore sections, as well as the main-production wellbore section. Reservoir-wellbore crossflow was analytically modeled for oil wells and gas wells. The following conclusions are drawn:

1. A comparison with data from two multilateral oil wells shows that the newly developed model is accurate for the purpose of engineering analyses.
2. The new model can be used for designing and evaluating performance of multilateral oil and gas wells.
3. Validation of the new model to apply to gas wells requires more data from multilateral gas wells.

Nomenclature

a_i = the base length of the triangular slice i, ft
A = characteristic area of the particle, ft^2
B = buoyant force, lb_f
d_c = particle equivalent diameter, in.
d_i = equivalent diameter of cutting size i, ft
d_s = solid particle equivalent diameter, in.
D_y = viscous drag force in the vertical direction, lb_f
E_k = kinetic energy per unit volume, $\text{lb}_\text{f}/\text{ft}^2$
g = 32.17 ft/s^2
g_c = $32.17\,\dfrac{\text{ft–lb}_\text{m}}{\text{lb}_\text{f}\text{–s}^2}$
h_i = height of slice i, ft
H = water depth, ft
m_c = mass of particle, lb_m
N_Re = Reynolds number
r_i = radius of the bottom area of cone, ft
T_i = thickness of slice i, ft.
v_f = current velocity, ft/s
v_x = velocity in the horizontal direction, ft/s
v_y = velocity in the vertical direction, ft/s
V_i = volume of slice i, ft^3
V_t = the total cuttings volume in the cuttings pile, ft^3
W = particle weight, lb_f
μ = viscosity, cp
ρ_c = particle density, $\text{lb}_\text{m}/\text{ft}^3$
ρ_f = fluid density, $\text{lb}_\text{m}/\text{ft}^3$
Ω = cuttings size distribution function

TABLE 6—RESERVOIR PROPERTIES AND WELLBORE CONFIGURATION (CASE A)

Horizontal Sections				
Lateral No.	1	2	3	
Reservoir pressure	2650	2608	2303	psia
Oil formation factor	1.03	1.02	1.02	STB/rb
Water formation factor	1.03	1.03	1.02	STB/rb
Bottomhole temp.	195	195	174	°F
Gas z-factor	0.90	0.90	0.90	
Gas specific gravity	0.60	0.60	0.60	air=1
Oil specific gravity	0.87	0.87	0.96	water=1
Water specific gravity	1.03	1.03	1.03	water=1
Water/oil ratio	0	0	0	stb/STB
Gas/oil ratio	500	500	500	scf/STB
Solution gas/oil ratio	50	50	50	scf/STB
Productivity index	85.5	99.8	2.32	STB/D/psi
Curvic Sections				
Lateral No.	1	2	3	
Radius of Curve	200	200	200	ft
Average inclination	45	45	45	deg.
Flow path diameter	5	5	5	in.
Flow path roughness	0.002	0.002	0.002	in.
Vertical Sections				
Lateral No.	1	2	3	
Interval length	45	510	4730	ft
Flow path diameter	5	5	5	in.
Flow path roughness	0.002	0.002	0.002	in.

Acknowledgment

The authors are grateful to Chevron and API for providing Board of Regents Chevron I&II Endowed Professorships and API Endowed Professorship in Petroleum Engineering throughout this study. This research is supported in part by the U.S. Dept. of Energy (DOE) under award number DE-FG02-04ER46136 and by the Board of Regents, State of Louisiana, under Contract No. DOE/LEQSF(2004-07)-ULL.

TABLE 7—MODEL-CALCULATED WELL AND LATERAL PRODUCTION RATES AT DIFFERENT WELLHEAD PRESSURES (CASE A)

Wellhead Pressure	Well	Layer 1	Layer 2	Layer 3
psi	STB/D	STB/D	STB/D	STB/D
0	103600	48536	54097	968
50	94217	44261	49104	852
100	84834	39986	44112	736
150	75450	35711	39119	620
200	66067	31436	34127	504
250	56683	27161	29134	388
300	47300	22886	24142	272
350	37916	18611	19149	156
400	28533	14336	14157	40
450	19149	10061	9164	–76
500	9766	5786	4172	–192
550	382	1511	-821	–308
552	7	1340	-1021	–312

TABLE 8—RESERVOIR PROPERTIES AND WELLBORE CONFIGURATION (CASE B)

Horizontal Sections					
Lateral No.	1	2	3	4	
Section length	115	90	136	165	ft
Layer permeability	6	8	11	13	md
Net pay thickness	17	18	15	14	ft
Reservoir pressure	2548	2430	2305	2117	psia
Radius of Drainage	1900	1850	2050	2080	ft
Gas viscosity	0.04	0.03	0.03	0.02	cp
Wellbore diameter	5	5	5	5	In.
Bottom temperature	188	186	183	180	°F
Gas z-Factor	0.87	0.91	0.96	0.97	
Gas specific gravity	0.85	0.83	0.8	0.75	air=1
Curvic Sections					
Lateral No.	1	2	3	4	
Radius of curve	225	217	234	270	ft
Average inclination	45	45	45	45	°
Flow path diameter	5	5	5	5	In.
Pipe roughness	0.0018	0.0018	0.0018	0.0018	In.
Vertical Sections					
Lateral No.	1	2	3	4	
Interval length	235	276	298	5046	ft
Flow path diameter	5	5	5	5	In.
Flow path roughness	0.0018	0.0018	0.0018	0.0018	In.

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TABLE 9—MODEL-CALCULATED WELL AND LATERAL PRODUCTION RATES AT DIFFERENT WELLHEAD PRESSURES (CASE B)

Wellhead Pressure psi	Well Mscf/d	Layer 1 Mscf/d	Layer 2 Mscf/d	Layer 3 Mscf/d	Layer 4 Mscf/d
0	39731	5994	9254	9928	14555
100	39628	5981	9233	9903	14511
200	39317	5943	9168	9826	14380
300	38799	5878	9059	9699	14163
400	38074	5787	8908	9521	13858
500	37142	5671	8713	9292	13466
600	36002	5528	8475	9013	12987
700	34656	5360	8193	8682	12421
800	33102	5166	7868	8300	11768
900	31341	4946	7500	7868	11027
1000	29373	4700	7088	7385	10200
1100	27198	4428	6634	6851	9286
1200	24815	4130	6135	6266	8284
1300	22225	3806	5594	5630	7196
1400	19429	3456	5009	4944	6020
1500	16425	3081	4381	4206	4757
1600	13213	2679	3709	3418	3407
1700	9795	2252	2995	2578	1970
1800	6170	1799	2237	1688	446
1900	2337	1320	1435	747	–1165
1958	19	1030	950	178	–2140
2000	-1703	815	590	-244	–2864

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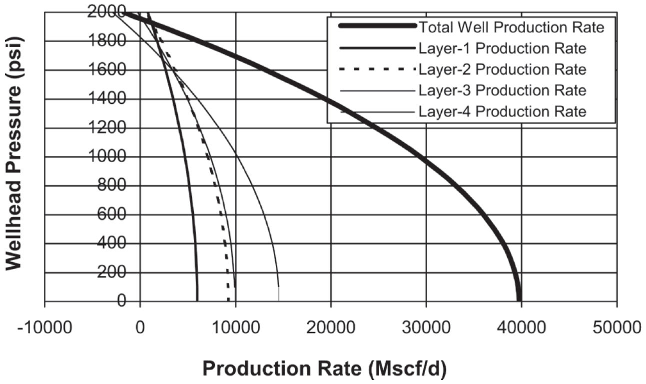

Fig. 4—Calculated performance of a multilateral gas well.

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SI Metric Conversion Factors

bbl × 1.589 873 E –01 = m^3
cp × 1.0* E–03 = Pa.s
ft × 3.048* E–01 = m
ft^3 × 2.831 685 E–02 = m^3
°F × (°F–32)/1.8 = °C
in. × 2.54* E+00 = cm
psi × 6.894 757 E–03 = MPa

*Conversion factors are exact.

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